

Virtual photons to fourth order CHPT with nucleons ^{#1}

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Baryon chiral perturbation theory offers another possibility of investigating isospin violation. As first stressed by Weinberg [1], reactions involving nucleons and neutral pions can lead to gross violations of isospin, e.g. in the scattering length difference $a(\pi^0 p) - a(\pi^0 n)$ he predicted an effect of the order of 30%. This is because chiral symmetry and isospin breaking appear at the same order and the leading isospin symmetric terms involving neutral pions are suppressed due to chiral symmetry. It is, however, known that precise and complete one loop calculations in the baryon sector should be carried out to fourth order since it has also been shown that in many cases one loop graphs with exactly one dimension two insertion are fairly large. Most calculations in baryon CHPT are performed in heavy baryon chiral perturbation theory (HBCHPT) [5, 6]. This is based on the observation that a straightforward extension of CHPT with baryons treated fully relativistically leads to a considerable complication in the power counting since only nucleon three-momenta are small compared to typical hadronic scales, as discussed in detail in ref.[7]. However, one has to be careful with strict non-relativistic expansions since in some cases they can conflict structures from analyticity, as discussed e.g. in ref.[8]. Therefore, in ref.[9], a novel scheme was proposed which is based on the relativistic formulation but circumvents the power counting problems (to one loop) by a clever separation of the loop integrals into IR singular and regular parts. In this formulation, all analytic constraints are fulfilled by construction.

The starting point of our approach is to construct the most general chiral invariant Lagrangian built from pions, nucleons and external scalar, pseudoscalar, vector, axial-vector sources *and* virtual photons, parametrized in terms of the vector field $A_\mu(x)$. The Goldstone bosons are collected in a 2×2 matrix-valued field $U(x) = u^2(x)$. The nucleons are described by structureless

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relativistic spin- $\frac{1}{2}$ particles, the spinors being denoted by $\Psi(x)$ in the relativistic case or by the so-called light component $N(x)$ in the heavy fermion formulation. The effective field theory admits a low energy expansion, i.e. the corresponding effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \dots, \quad (1)$$

where the ellipsis denotes terms of higher order not considered here. For the explicit form of the meson Lagrangian and the dimension one and two pion–nucleon terms, we refer to ref.[11]. More precisely, in the pion–nucleon sector, the inclusion of the virtual photons modifies the leading term of dimension one and leads to new local (contact) terms starting at second order [2]. In particular, since the electric charge related to the virtual photons always appears quadratic, the following pattern for the terms in the electromagnetic effective Lagrangian emerges. At second order, we can only have terms of order e^2 , at third order $e^2 q$ and at fourth order $e^2 q^2$ or e^4 (besides the standard strong terms). The inclusion of the virtual photons proceeds with,

$$Q_{\pm} = \frac{1}{2} \left(u Q u^{\dagger} \pm u^{\dagger} Q u \right), \quad (2)$$

which under chiral $\text{SU}(2)_L \times \text{SU}(2)_R$ symmetry transform as any matrix-valued matter field (Q defines the charge matrix).

In particular, to lowest order one finds (in the relativistic and the heavy fermion formulation)

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\gamma_{\mu} \cdot \tilde{\nabla}^{\mu} - m + \frac{1}{2} g_A \gamma^{\mu} \gamma_5 \cdot \tilde{u}_{\mu} \right) \Psi = \bar{N} \left(i v \cdot \tilde{\nabla} + g_A S \cdot \tilde{u} \right) N + \mathcal{O}\left(\frac{1}{m}\right), \quad (3)$$

with

$$\tilde{\nabla}_{\mu} = \nabla_{\mu} - i Q_{+} A_{\mu}, \quad \tilde{u}_{\mu} = u_{\mu} - 2 Q_{-} A_{\mu}, \quad (4)$$

and^{#3}

$$\Psi(x) = \exp\{i m v \cdot x\} (N(x) + h(x)). \quad (5)$$

Furthermore, v_{μ} denotes the nucleons' four-velocity, S_{μ} the covariant spin-vector à la Pauli–Lubanski and g_A the axial-vector coupling constant. These virtual photon effects can only come in via loop diagrams since by definition a virtual photon can not be an asymptotic state.

At second order, local contact terms with finite low-energy constants (LECs) appear. We call these LECs f_i for the heavy baryon approach and f'_i in the relativistic Lagrangian. As stated before, the em Lagrangian is given entirely in terms of squares of Q_{\pm} (and their traceless companions),

$$\mathcal{L}_{\pi N, \text{em}}^{(2)} = \sum_{i=1}^3 F_{\pi}^2 f'_i \bar{\Psi} \mathcal{O}_i^{(2)} \Psi = \sum_{i=1}^3 F_{\pi}^2 f_i \bar{N} \mathcal{O}_i^{(2)} N, \quad (6)$$

with the $\mathcal{O}_i^{(2)}$ monomials of dimension two,

$$\mathcal{O}_1^{(2)} = \langle \tilde{Q}_+^2 - \tilde{Q}_-^2 \rangle, \quad \mathcal{O}_2^{(2)} = \langle Q_+ \rangle \tilde{Q}_+, \quad \mathcal{O}_3^{(2)} = \langle \tilde{Q}_+^2 + \tilde{Q}_-^2 \rangle. \quad (7)$$

^{#3}We do not spell out the details of how to construct the heavy nucleon EFT from its relativistic counterpart but refer the reader to the extensive review [11].

Notice furthermore that only the second term in eq.(6) has an isovector piece and contributes to the neutron–proton mass difference [2]. The factor F_π^2 in eq.(6) ensures that the em LECs have the same dimension as the corresponding strong dimension two LECs. From the third order calculation of the proton–neutron mass difference [2] one deduces the value for f_2 , $f_2 = -(0.45 \pm 0.19) \text{ GeV}^{-1}$.

The Lagrangian to third order takes the form

$$\mathcal{L}_{\pi N, \text{em}}^{(3)} = \sum_{i=1}^{12} F_\pi^2 g'_i \bar{\Psi} \mathcal{O}_i^{(3)} \Psi = \sum_{i=1}^{12} F_\pi^2 g_i \bar{N} \mathcal{O}_i^{(3)} N, \quad (8)$$

with the $\mathcal{O}_i^{(3)}$ monomials in the fields of dimension three [2][3], in their relativistic form and the heavy baryon counterparts. Again, for the g_i to have the same mass dimension as the d_i of the strong sector defined in ref.[10], we have multiplied them with a factor of F_π^2 . Thus the g_i (g'_i) scales as $[\text{mass}^{-2}]$. So the complete fourth order pion–nucleon Lagrangian with virtual photons is given by [3]

$$\mathcal{L}_{\pi N, \text{em}}^{(4)} = \sum_{i=1}^5 F_\pi^4 h'_i \bar{\Psi} \mathcal{O}_i^{(e^4)} \Psi + \sum_{i=6}^{90} F_\pi^2 h'_i \bar{\Psi} \mathcal{O}_i^{(e^2 p^2)} \Psi \quad (9)$$

with the $\mathcal{O}_i^{(4)}$ monomials in the fields of dimension four [3]. To be consistent with the scaling properties of the dimension two and three LECs, the h_i are multiplied with powers of F_π^2 such that the first five LECs take dimension $[\text{mass}^{-3}]$ while the others are of dimension $[\text{mass}^{-1}]$.

The scalar form factor

The scalar form factor of the nucleon is defined via

$$\langle N(p') | m_u \bar{u}u + m_d \bar{d}d | N(p) \rangle = \bar{u}(p') u(p) \sigma(t), \quad t = (p' - p)^2, \quad (10)$$

for a nucleon state $|N(p)\rangle$ of four–momentum p . At $t = 0$, which gives the much discussed pion–nucleon σ –term, one can relate this matrix element to the so–called strangeness content of the nucleon. A direct determination of the σ –term is not possible, but rather one extends pion–nucleon scattering data into the unphysical region and determines $\sigma_{\pi N}(t = 2M_\pi^2)$, i.e. at the so–called Cheng–Dashen point. The relation to the σ –term is given by the low–energy theorem of Brown, Peccei and Pardee [13],

$$\sigma_{\pi N}(2M_\pi^2) = \sigma_{\pi N}(0) + \Delta\sigma_{\pi N} + \Delta R \quad (11)$$

where $\Delta\sigma_{\pi N}$ parametrizes the t –dependence of the sigma–term whereas ΔR is a remainder not fixed by chiral symmetry. The most systematic determination of $\Delta\sigma_{\pi N}$ has been given in ref.[14], $\Delta\sigma_{\pi N} = (15 \pm 1) \text{ MeV}$. The remainder ΔR has been bounded in ref.[15], $\Delta R < 2 \text{ MeV}$. It was shown that the third order effects can shift the proton σ –term by about 8% and have a smaller influence on the shift to the Cheng–Dashen (CD) point [2]. In [3] we worked out explicitly the isospin violating corrections to this shift to fourth order. This is motivated by the fact that in the difference most of the counterterm contributions drop out, more precisely, only momentum–dependent contact terms can contribute to the shift. Such terms only appear at fourth order since

due to parity one needs two derivatives and any quark mass or em insertion accounts for at least two orders [3]. It can be decomposed as

$$\sigma_{\pi N}^{(4)}(t) = \sigma_{\pi N}^{(4),\text{IC}}(t) + \sigma_{\pi N}^{(4),\text{IV}}(t) . \quad (12)$$

The isospin-conserving strong terms have already been evaluated in ref.[15]. Here, we concentrate on the em corrections ~ 1 (in isospin space) and all terms $\sim \tau_3$. We have eye graphs and tadpoles with insertions $\sim f_2, c_5$. These can be evaluated straightforwardly. Because of the tiny coefficients appearing in the evaluation of the loop contributions, these are only fractions of an MeV, $\Delta\sigma_{\pi N}^{4,\text{IV},\text{loop}} = -0.05 \text{ MeV}$ and can thus be completely neglected. For the counterterm contributions, setting all appearing LECs on the values obtained from dimensional analysis as explained [3], one finds a total contribution $\Delta\sigma_{\pi N}^{4,\text{IV},\text{ct}} = \pm 0.01 \text{ MeV}$. For the IC em terms, we find (setting again $f_{1,3} = \pm 1/4\pi$) a completely negligible loop contribution (less than 0.01 MeV) and the counterterms give $\pm 0.7 \text{ MeV}$ for the LECs estimated from dimensional analysis. Note, however, that if the numerical factors $f_{1,3}/(4\pi)$ are somewhat bigger than one, one could easily have a shift of $\pm 2 \text{ MeV}$, which is a substantial electromagnetic effect.

Neutral pion scattering off nucleons

As pointed out long time ago by Weinberg [1], the difference in the S-wave scattering lengths for neutral pions off nucleons is sensitive to the light quark mass difference,

$$\begin{aligned} a(\pi^0 p) - a(\pi^0 n) &= \frac{1}{4\pi} \frac{1}{1 + M_{\pi^+}/m_p} \frac{-4B(m_u - m_d)c_5}{F_\pi^2} + \mathcal{O}(q^3) \\ &= \frac{1}{4\pi} \frac{1}{1 + M_{\pi^+}/m_p} \Delta_2(M_{\pi^0}) + \mathcal{O}(q^3) . \end{aligned} \quad (13)$$

It was shown in ref.[2] by an explicit calculation that to third order there are no corrections to this formula. This is based on the fact that the electromagnetic Lagrangian can not contribute at this order since the charge matrix has to appear quadratic and never two additional pions can appear. However, at next order one can of course have loop graphs with one dimension two insertion and additional em counterterms. To obtain the first correction to Weinberg's prediction, eq.(13), one thus has to compute the fourth order corrections [3]. These are due to strong dimension two insertions $\sim c_5$ and em insertions $\sim f_2$. For the difference $a(\pi^0 p) - a(\pi^0 n)$ we only have to consider the operators $\sim \tau^3$ [3]. Consider first the loop contributions. Since we can not fix the counterterms from data, we are left with a spurious scale dependence which reflects the theoretical uncertainty at this order. For $\lambda = \{0.5, 0.77, 1.0\} \text{ GeV}$ we find

$$\Delta_2^{\text{str}} = \{-7.1, 0.9, 5.7\} \cdot 10^{-2} , \quad \Delta_2^{\text{em}} = \{11.5, 12.0, 12.3\} \cdot 10^{-2} . \quad (14)$$

The counterterms are estimated based on dimensional analysis at the scale $\lambda = M_\rho$ and give a contribution of about $-0.3 \cdot 10^{-2}$. Even if the LECs would be a factor of ten larger than assumed, the counterterm contribution would not exceed $\pm 3\%$. Altogether, the correction to Weinberg's prediction, eq.(13), are in the range of 4 to 18 percent, i.e. fairly small. Finally, we wish to mention that in ref.[16] isospin-violation for neutral pion photoproduction off nucleons was discussed which allows one to eventually measure directly the very small $\pi^0 p$ scattering length by use of the final-state theorem.

Summary

We have developed the whole mechanism to calculate isospin violation effects in the framework of CHPT. In order to get the correct size of isospin violation one has to include all non-isospin symmetric sources like the electromagnetic interaction, the quark mass difference ($\pi_0 - \eta$ mixing), explicit photon loops. In future calculations one has to pin down the two LECs of the second order em Lagrangian $f_{1,3}$ which are not well known and are estimated only by dimensional arguments.

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